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Distributed multiple model joint probabilistic data association with Gibbs sampling-aided implementation

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A B S T R A C T

This paper proposes a new distributed multiple model multiple manoeuvring target tracking algorithm. The proposed tracker is derived by combining joint probabilistic data association (JPDA) with consensus-based distributed filtering. Exact implementation of the JPDA involves enumerating all possible joint association events and thus often becomes computationally intractable in practice. We propose a computationally tractable approximation of calculating the marginal association probabilities for measurement-target mappings based on stochastic Gibbs sampling. In order to achieve scalability for a large number of sensors and high tolerance to sensor failure, a simple average consensus algorithm-based information JPDA filter is proposed for distributed tracking of multiple manoeuvring targets. In the proposed framework, the state of each target is updated using consensus-based information fusion while the manoeuvre mode probability of each target is corrected with measurement probability fusion. Simulations clearly demonstrate the effectiveness and characteristics of the proposed algorithm. The results reveal that the proposed formulation is scalable and much more efficient than classical JPDA without sacrificing tracking accuracy.

1. Introduction

Proliferation of low-cost, lightweight, and power efficient sensors and advances in networked systems enable the employment of multiple sensor nodes, capable of communicating with each other. The sensors in a network cooperatively enable complicated sensing and tracking tasks, which are otherwise difficult to accomplish. Compared to the single sensor target tracking, utilising multiple sensors, through information fusion, can significantly improve the sensor coverage and the estimation accuracy [1]. The challenge is that these sensors are likely to contain some degree of uncertainties. Low-cost sensors are generally subject to high clutter rate and low detection probability. Combined with the inherent uncertainties and complexity of the problem, the poor performance issue with these sensors could be significantly exacerbated in target tracking, especially in multi-target tracking [2–4]. When targets are manoeuvring, the problem becomes even more challenging. Practical applications that involve manoeuvring targets include, but are not limited to, aircraft tracking, ground moving vehicle tracking, re-entry vehicle tracking, and human tracking. However, algorithms for multiple manoeuvring targets tracking in a sensor network are rare. Therefore, it is meaningful to develop a tractable multi-sensor multiple manoeuvring targets tracking algorithm.

The objective of this paper is, in fact, to address the problem of distributed multiple manoeuvring targets tracking in a sensor network, subject to a certain degree of uncertainties. Generally, the multi-sensor multi-target tracking is divided into two stages: the first stage is a local multi-target tracking phase and the second is the estimation fusion among all sensors. The focus of this paper is the development of efficient algorithms for handling important issues in both stages.

In the local estimation stage, each sensor node runs a multi-target tracking (MTT) algorithm to obtain the local tracks. As discussed, the key issue is that the measurement uncertainty could significantly degrade the performance of MTT. Data association is a plausible and widely-accepted solution in multi-target tracking to resolve the problem of measurement uncertainty. This technique discerns target-generated measurements from clutters and finds the mappings between targets and measurements. One of the most well-known association algorithms is the multiple hypothesis tracking (MHT) [5,6]. MHT solves the problem of association ambiguity by a delayed logic, which maintains all data association hypotheses in a decision-making tree unless enough information is available to remove the impossible hypothesis. Although MHT is proved to be Bayesian optimal for MTT, finding the exact solution is computationally intractable and hence requires approximated implementations[7–10]. Another widely-accepted probabilistic data association approach, joint probabilistic data association (JPDA), is known as a suboptimal MTT estimator that can achieve reasonable results at lower computational burden [11].

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Considering the balance between the sensitivity issue and the computational cost, this paper adopts JPDA as the underlying data association approach. The issue with JPDA is that it requires to enumerate all feasible joint events to find the marginal association probability. Note that the marginal probability is then used to perform moment matching to merge the posterior Gaussian mixture into a single Gaussian form. Computing the marginal probabilities itself is #P-complete, thereby leading to intractability for a large number of targets. To reduce the computational cost of the JPDA, many *ad hoc* formulations and approximations have been developed, [12–15]. Unlike previous accountable approximations, this paper proposes a new stochastic sampling-based implementation of JPDA that greatly improves the computational efficiency and maintains the robustness of standard JPDA against outliers. Note that these characteristics are of great importance for sensor networks, where the computational power is limited.

In the fusion stage, the sensor nodes communicate with each other to perform estimation fusion through a network topology. Unlike the centralised filter, the distributed estimation is known to exhibit the advantages of scalability for large-scale networks and strong robustness against sensor fault [16–19]. Among the existing distributed approaches, consensus-based methods [19–21] are widely-used due to their global convergence and easy implementation. By iteratively communicating with the adjacent sensors at each time instant, the estimation obtained by each sensor asymptotically converges to the global one. The Kalman consensus filter (KCF) [16,19,22] is a well-known distributed filter, which directly applies the average consensus algorithm to local state estimations. However, it only works well in the situation where each sensor can get a measurement from the target [23,24]. In [25,26], a distributed multi-sensor multi-target tracking algorithm was proposed on the basis of KCF and thus cannot account for the naive sensors,[1] e.g., targets are out of sensors' field-of-view [24]. In a recent notable contribution [24], the authors proposed an information consensus filter (ICF) that addresses the inherent problems of KCF and guarantees convergence to the centralised one. Based on either KCF or ICF, different distributed estimation algorithms for single manoeuvring target tracking were proposed in [27–29]. Note that although distributed multi-sensor using consensus algorithm for single target tracking is well-established, direct extension to the MTT scenario is unreasonable due to the measurement origin uncertainty and therefore requires careful adjustment. Combining the probabilistic data association (PDA) filter [30] with the idea of ICF, a multi-sensor multi-target tracking filter was developed in [31] for a sensor network, but this algorithm was shown to be sensitive to clutter rate. By incorporating consensus algorithm with Probability Hypothesis Density (PHD) filter, the authors in [32,33] proposed novel multi-sensor multi-target tracking approaches. Note that these two distributed PHD filters are based on classical geometric average fusion rule. The authors in [34] first developed a new consensus-based PHD filter by utilising the arithmetic average fusion rule. It has been later demonstrated that the simpler arithmetic average fusion outperforms the geometric average fusion in some cases [35,36]. The consensus-based PHD filters, however, can not preserve track continuity, e.g., cannot provide target identity information. This issue was later resolved by the consensus-based [37,38] δ-Generalised Labeled Multi-Bernoulli (δ-GLMB), which shares similar concept as classical data association techniques in the implementation. The authors in [39–41] proposed two multi-sensor multi-target tracking filters: parallel and sequential multi-sensor JPDAs. The parallel version was shown to be exponentially computationally complex as the total number of sensors increases [39]. On the other hand, the sequential one has lower survivability [40], i.e., it requires each sensor's field-of-view to cover the entire surveillance region. These approaches, however, require sequentially connected sensor networks and are not really distributed trackers.

Motivated by the above observations, this paper aims to develop a tractable/practical algorithm that is suitable for multiple manoeuvring targets tracking using a partially connected sensor network. The main contributions of this paper are highlighted as follows:

(1) An efficient algorithm for the JPDA implementation is proposed by utilising stochastic Gibbs sampling. Each possible joint event is considered as a random variable that can be generated by stochastic Gibbs sampling and hence the marginal association probability can be easily approximated by the event occurrence. This polynomial-time approximation makes it feasible to apply JPDA in a sensor network for multi-target tracking. Experiments show that the proposed approximation is scalable and of great efficiency with ignorable performance sacrifice.

(2) A general framework of distributed multiple sensors multiple manoeuvring targets tracking algorithm is developed by incorporating the consensus algorithm and interactive multiple model (IMM) approach into the proposed Gibbs-JPDA filter. More specifically, we formulate the state estimation of each manoeuvre mode in the form of information state fusion by developing a distributed information consensus JPDA filter. Based on the jump Markov nonlinear system (JMNS) modelling, the proposed general framework incorporates the IMM filter by using a distributed measurement probability fusion scheme to provide the capability of accurate estimation for manoeuvring targets. Due to the distributed nature, the proposed algorithm has strong robustness against sensor failures.

Note that the tracking algorithm developed has already been applied and tested in the *EuroSwarm*[2] project. The corresponding indoor demo is also attached in the Supplementary file.

The rest of the paper is organised as follows. Section 2 presents some preliminaries and backgrounds. Section 3 provides the details of the proposed Gibbs sampling-aided marginalisation. In Section 4, the UKF-based distributed information JPDA filter is derived in detail, followed by the proposed multiple model UKF-based distributed information JPDA filter shown in Section 5. Finally, some simulation results and conclusions are offered.

2. Backgrounds and preliminaries

This section first provides some necessary backgrounds of the basics of JMNS, to facilitate the analysis in the following sections. Then, the problem formulation of the paper is stated.

2.1. Multiple-target jump markov nonlinear system

Let $X_k = \left\{ x_k^1, \dots, x_k^{N_k} \right\}$ be the set of target states at scan k, where N_k denotes the number of targets at scan k, x_k^i the ith target at scan k. The target considered in this paper manoeuvres according to various kinematic models. In the case of manoeuvring target estimation, one key issue is how to construct a suitable model to represent the system transition model. To date, the most widely-accepted idea is the JMNS modelling [42,43], which assumes that the target motion can be quantified by a weighted sum of several manoeuvre modes. A JMNS consists of a set of different nonlinear models and each model is quantified by its mode probability. The mode probability evolves with time according to a finite state Markov chain and determines how probable the manoeuvre mode follows the real target motion model. Under the JMNS framework, the ith target can be modeled by the following discrete-time jump Markov nonlinear system

$$x_k^i = f^i\left(x_{k-1}^i, r_k^i\right) + \omega_{k-1}^i\left(r_k^i\right) \tag{1}$$

where f^i denotes the system dynamics transition function of the ith target, r_k^i the target manoeuvre mode, and $\omega_{k-1}^i\left(r_k^i\right)$ the process noise. We assume that r_k^i takes value from a finite set $\Xi = \left\{1, 2, \dots, \mathcal{M}_r\right\}$

[1] Naive sensors mean the sensors that cannot detect the same target as other sensors.

[2] Description can be accessed through https://www.cranfield.ac.uk/research-projects/euroswarm-developing-technology-for-uav.

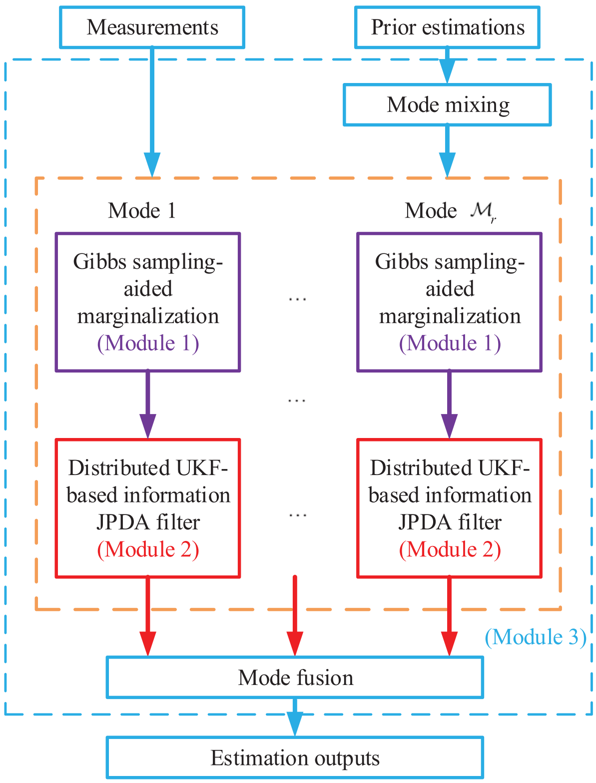

Fig. 1. General information flow of the proposed algorithm.

with mode transition probability matrix $\Pi = [\pi_{ms}]_{\mathcal{M}_r \times \mathcal{M}_r}$, where $\pi_{ms} \triangleq \Pr\left\{ r^i_{k+1} = s \middle| r^i_k = m \right\}$ for all m, $s \in \Xi$ and $\sum_{s=1}^{\mathcal{M}_r} \pi_{ms} = 1$ for any $m \in \Xi$. The Markovian transition probability matrix determines the possibility of a certain motion model that the target follows in the next scan $k+1$ given a motion model at current scan k.

2.2. Problem formulation

The aim of this paper is to design a distributed multiple manoeuvring targets (e.g., multiple JMNSs) tracking algorithm using a partially connected sensor network. Each sensor node runs a local multi-target tracking algorithm and the local estimations are then fused in a distributed way. Note that in a partially or not fully connected sensor network, each sensor can only communicate with its neighbours. The main challenges of the considered problem are twofold. On one hand, most multi-target tracking algorithms have high computational burden and thus have limited applications in multi-sensor network systems. Although the JPDA can achieve reasonable accuracy with less computational power than the MHT, full enumeration of all possible joint events is still intractable for practical applications. On the other hand, the posterior of multi-target estimations contain the measurement origin uncertainty, meaning that direct extension of distributed single manoeuvring target multi-sensor fusion algorithms to the multiple manoeuvring targets case is not feasible and thus requires careful adjustment.

In order to tackle these challenges, we tailor a framework incorporating the information-form JPDA filter with IMM for multi-sensor multiple manoeuvring targets tracking. The proposed framework is illustrated in Fig. 1. In the local estimation, we consider each joint association event in JPDA as a random variable that satisfies a distribution and then propose to leverage the stochastic Gibbs sampling to calculate the approximated marginal probability of JPDA filter. This stochastic approximation can significantly reduce the computational burden and retain the properties of the original JPDA. For estimation fusion, we reformulate the JPDA filter in an information form and then utilise the average consensus algorithm for both target state fusion and model probability fusion in a distributed way. In summary, the proposed tracking algorithm consists of three modules: (1) Gibbs sampling based JPDA (Section 3); (2) distributed UKF-based information JPDA filter for each manoeuvre mode (Section 4); and (3) multiple model estimation fusion (Section 5).

3. Gibbs sampling based JPDA filter

This section proposes a new JPDA filter algorithm using a stochastic Gibbs sampling approach. For brevity, we ignore the sensor index as well as the mode index here.

3.1. Standard JPDA filter

Let us briefly review the classical JPDA filter for the completeness of the paper. The set of measurements received by one sensor at scan k is defined as $Z_k = \left\{ z_{0,k}, z_{1,k}, \dots, z_{M_k,k} \right\}$, where M_k denotes the number of measurements received at scan k, $z_{j,k} (j \neq 0)$ the jth measurement received at scan k, $z_{0,k}$ the dummy measurement for convenient representation of miss detection and false alarm. The number of clutters or false alarms is assumed to be a Poisson distribution and λ_F denotes the expected number of clutters per unit volume of the validation gate, known as spatial density of clutters.

In the standard JPDA filtering approach, each measurement is assumed to originate from a number of candidate targets and thus tracks are updated by a weighted sum of the validated measurements from the current time. This is the reason why JPDA is known as a 'soft decision' filter. This feature makes the posterior probability distribution a Gaussian mixture form. Propagation of the Gaussian mixture distribution over time contains an exponential number of mixture components and is thus intractable without approximations. In order to maintain the feasibility, JPDA uses a single Gaussian model to approximate the Gaussian mixture at each time step. More specifically, the state estimation is obtained by using a weighted innovation term

$$\tilde{z}^i_k = \sum_{j=1}^{M_k} \beta^i_j \tilde{z}^i_{j,k} = \sum_{j=1}^{M_k} \beta^i_j \left(z_{j,k} - \hat{z}^i_k \right) \tag{2}$$

where $\hat{z}^i_k$ denotes the predicted measurement of the ith target, β^i_j the marginal association probability that the jth measurement is associated with the ith target.

It is clear that determining the marginal association probabilities β^i_j is the key part of JPDA. JPDA algorithm calculates the marginalised association probability based on all possible joint association events. A feasible joint event is defined as one possible mapping of the measurements to the tracks such that: (1) each measurement (except for the dummy one) is assigned to at most one target; (2) each target is uniquely assigned to a measurement. Let $\Theta_k = \left\{ \theta^i_k \right\}$, $i \in \left\{ 1, 2, \dots, N_k \right\}$, denote the joint association event at scan k, where $\theta^i_k \in \left\{ 0, 1, \dots, M_k \right\}$ stands for the single association event. Here, $\theta^i_k = j$ means that the jth measurement originates from the ith target. The posterior distribution of the joint event is

$$p\left(\Theta_k \middle| Z_k\right) \propto \left(\prod_{i=1}^{N_k} \varphi_i\left(\theta^i_k\right) \right) \left(\prod_{(i,i') \in E} \varphi_c\left(\theta^i_k, \theta^{i'}_k\right) \right) \tag{3}$$

where $\varphi_i\left(\theta^i_k\right)$ denotes the un-normalised PDA probability of event θ^i_k given by [30]

$$\varphi_i\left(\theta^i_k = j\right) \propto \begin{cases} \left(1 - P_D P_G\right) \lambda_F, & j = 0 \\ \mathcal{N}\left(z_{j,k}; H^i_k x^i_k, S^i_k\right) P_D, & j \neq 0 \end{cases} \tag{4}$$

where $\mathcal{N}(x; \mu, \Sigma)$ denotes the Gaussian distribution of variable x with mean μ and covariance Σ, H^i_k the measurement matrix, S^i_k the innovation covariance matrix, P_D the probability of detection, P_G the gating probability, and

$$\varphi_c\left(\theta^i_k, \theta^{i'}_k\right) = \begin{cases} 0, & \theta^i_k = \theta^{i'}_k > 0 \\ 1, & \text{otherwise} \end{cases} \tag{5}$$

$$E = \left\{ (i, i') \middle| \exists i \in [N_k], i' \in [N_k], i \neq i' \right\} \tag{6}$$

which ensure that one measurement (except for the dummy measurement) can only be allocated to one target.

The marginalised association probability β_j^i indicating that the jth measurement is associated with the ith target can be obtained by the law of total probability as

$$\beta_j^i = \sum_{\Theta_k : \theta_k^i = j} p(\Theta_k | Z_k) \tag{7}$$

which implies that exact solution of the marginal association probability requires fully enumerating all possible joint events.

Remark 1. Note that the standard JPDA filter requires the assumption that the number of targets is known a priori [11]. The same assumption is also utilised in this paper. However, this algorithm can be adapted to more practical scenarios, where the number of targets is unknown, by either simple heuristic *M/N* logic [6] or birth model approach [44].

3.2. Gibbs sampling-aided marginalisation

Determining the marginal joint association probabilities β_j^i between measurements and targets, which is a well-known #P-complete problem. To tackle with the combinatorial nature in obtaining the association probability, this paper proposes a sampling based, specifically Gibbs sampling-based, algorithm.

The key idea that this paper proposes is to consider each joint association event Θ_k as a random variable that satisfies a distribution $\pi(\Theta_k)$. To this end, we construct a Markov chain whose state space is the set of all feasible joint events with stationary distribution as the posterior joint event distribution. To ensure that the joint events with higher probability are more easily to be sampled, it is natural to construct the sampling proposal $\pi(\Theta_k)$ proportional to its corresponding joint posterior probability, i.e.,

$$\pi(\Theta_k) \propto \left(\prod_{i=1}^{N_k} \varphi_i(\theta_k^i) \right) \left(\prod_{(i,i') \in E} \varphi_c(\theta_k^i, \theta_k^{i'}) \right) \tag{8}$$

Using the sampling proposal $\pi(\Theta_k)$, we could generate sufficiently enough samples of Θ_k. From the samples, it is straightforward to approximate the marginal joint association probability by the sample occurrence.

The issue is that direct sampling from (8) is very difficult as enumerating all possible joint events is impossible for real-time applications. Therefore, we develop a sampling-based marginalisation algorithm using Gibbs sampling. Gibbs sampling is a stochastic method for Bayesian inference to approximate the posterior multivariate probability distribution in a polynomial time [45,46]. This sampling approach was also utilised in the implementation of δ-GLMB in [47]. The main advantage is that it is simpler to recursively sample from a conditional distribution than to sample directly from the joint distribution itself. More specifically, the transition kernel from one joint event $\Theta_k = \left(\theta_k^1, \ldots, \theta_k^{N_k}\right)$ to another joint event $\bar{\Theta}_k = \left(\bar{\theta}_k^1, \ldots, \bar{\theta}_k^{N_k}\right)$ is given by

$$\pi(\bar{\Theta}_k | \Theta_k) = \prod_{m=1}^{N_k} \pi_m\left(\bar{\theta}_k^m \middle| \bar{\theta}_k^1, \ldots, \bar{\theta}_k^{m-1}, \theta_k^{m+1}, \ldots, \theta_k^{N_k}\right) \tag{9}$$

where π_m can be obtained from (4) and (8) as

$$\begin{aligned}
&\pi_m\left(\bar{\theta}_k^m \middle| \bar{\theta}_k^1, \ldots, \bar{\theta}_k^{m-1}, \theta_k^{m+1}, \ldots, \theta_k^{N_k}\right) \\
&= \frac{\pi\left(\bar{\theta}_k^1, \ldots, \bar{\theta}_k^m, \theta_k^{m+1}, \ldots, \theta_k^{N_k}\right)}{\pi\left(\bar{\theta}_k^1, \ldots, \bar{\theta}_k^{m-1}, \theta_k^{m+1}, \ldots, \theta_k^{N_k}\right)} \\
&\propto \pi\left(\bar{\theta}_k^1, \ldots, \bar{\theta}_k^m, \theta_k^{m+1}, \ldots, \theta_k^{N_k}\right) \\
&= \left(\varphi_m(\vartheta_k^m) \prod_{(m,i') \in E} \varphi_c\left(\vartheta_k^m, \vartheta_k^{i'}\right) \right) \\
&\quad \times \left(\prod_{i \neq m} \varphi_i(\vartheta_k^i) \prod_{(i,i') \in E, i \neq m} \varphi_c\left(\vartheta_k^i, \vartheta_k^{i'}\right) \right) \\
&\propto \varphi_m(\vartheta_k^m) \prod_{(m,i') \in E} \varphi_c\left(\vartheta_k^m, \vartheta_k^{i'}\right)
\end{aligned} \tag{10}$$

where $\vartheta_k^i = \bar{\theta}_k^i, i \in \{1, \ldots, m\}$, $\vartheta_k^i = \theta_k^i, i \in \{m+1, \ldots, N_k\}$

'Proportion to' in Eq. (10) highlights the dependence of individual conditional distribution on ϑ_k^m, while rest parts are formed as the normalisation constant. Given the joint event Θ_k, a joint event $\bar{\Theta}_k$ can be obtained by recursive sampling according to the following individual conditional distributions

$$\begin{aligned}
\bar{\theta}_k^1 &\sim \pi_1\left(\bar{\theta}_k^1 \middle| \theta_k^2, \ldots, \theta_k^{N_k}\right) \\
&\vdots \\
\bar{\theta}_k^m &\sim \pi_m\left(\bar{\theta}_k^m \middle| \bar{\theta}_k^1, \ldots, \bar{\theta}_k^{m-1}, \theta_k^{m+1}, \ldots, \theta_k^{N_k}\right) \\
&\vdots \\
\bar{\theta}_k^{N_k} &\sim \pi_{N_k}\left(\bar{\theta}_k^{N_k} \middle| \bar{\theta}_k^1, \ldots, \bar{\theta}_k^{N_k-1}\right)
\end{aligned} \tag{11}$$

Once an enough number of samples generated, the marginal joint association probability is approximated by occurrence. After constructing the Markov chain, it is necessary to prove that the generated Markov chain asymptotically converges to its invariant distribution and this property is formulated in Theorem 1.

Theorem 1. *Given any initial feasible joint event, the distribution of Gibbs samples (9) asymptotically converges to the target distribution (8) with an exponential rate as*

$$\left| \pi^n(\bar{\Theta}_k | \Theta_k) - \pi(\bar{\Theta}_k) \right| \le (1 - 2\beta)^{\lfloor n/2 \rfloor} \tag{12}$$

where $\pi^n(\bar{\Theta}_k | \Theta_k)$ *denotes the nth power of transition kernel* $\pi(\bar{\Theta}_k | \Theta_k)$, $\beta = \min_{\Theta_k} \pi^2(\bar{\Theta}_k | \Theta_k) \in (0, 0.5]$ *the least likely two-step transition probability.*

Proof. In general, the convergence of finite-state Markov chain is guaranteed by its irreducibility and regularity. The irreducibility of a Markov chain is quantified in terms of the possibility that one state has capability to transfer to another state within finite step. And the regularity of a Markov chain can be checked by the positivity of the entries of some finite power of its transition matrix.

Let 0_n denote the n dimensional zero vector. Since every target can share the dummy measurement, i.e., $\varphi_c(0, \theta_k^i) = 1$, it follows from (10) that

$$\begin{aligned}
\pi\left(0_{N_k} | \Theta_k\right) &\propto \prod_{m=1}^{N_k} \varphi_m(0) > 0 \\
\pi\left(\Theta_k \middle| 0_{N_k}\right) &\propto \prod_{m=1}^{N_k} \varphi_m(\theta_k^m) > 0
\end{aligned} \tag{13}$$

Then, the two-step transition kernel from any θ to any $\bar{\theta}$ satisfies

$$\begin{aligned}
\pi^2(\bar{\Theta}_k | \Theta_k) &= \sum_{\zeta} \pi(\bar{\Theta}_k | \zeta) \pi(\zeta | \Theta_k) \\
&> \pi\left(\bar{\Theta}_k \middle| 0_{N_k}\right) \pi\left(0_{N_k} | \Theta_k\right) > 0
\end{aligned} \tag{14}$$

This implies that the Markov chain $\left\{\Theta_k^{(t)}\right\}_{t=1}^{\infty}$ generated by the Gibbs sampler is irreducible and recurrent, and therefore the Markov chain will asymptotically converge to its invariant distribution, e.g., the posterior of the joint event, by the ergodic theorem [48]. Next, applying Lemma 2, presented in Appendix A, to $\pi^2(\bar{\Theta}_k | \Theta_k)$ gives

$$\begin{aligned}
&\max_{\Theta_k} \pi^{2n}(\bar{\Theta}_k | \Theta_k) - \min_{\Theta_k} \pi^{2n}(\bar{\Theta}_k | \Theta_k) \le (1 - 2\beta)^n \\
&\lim_{n \to \infty} \max_{\Theta_k} \pi^{2n}(\bar{\Theta}_k | \Theta_k) = \lim_{n \to \infty} \min_{\Theta_k} \pi^{2n}(\bar{\Theta}_k | \Theta_k) \ge \beta > 0
\end{aligned} \tag{15}$$

Since Lemma 1, presented in Appendix A, states that $\max_{\Theta_k} \pi^n(\bar{\Theta}_k|\Theta_k)$ is non-increasing and $\min_{\Theta_k} \pi^n(\bar{\Theta}_k|\Theta_k)$ is non-decreasing in n, (15) can be reformulated as

$$\begin{aligned} &\max_{\Theta_k} \pi^n(\bar{\Theta}_k|\Theta_k) - \min_{\Theta_k} \pi^n(\bar{\Theta}_k|\Theta_k) \le (1-2\beta)^{\lfloor n/2 \rfloor} \\ &\lim_{n\to\infty} \max_{\Theta_k} \pi^n(\bar{\Theta}_k|\Theta_k) = \lim_{n\to\infty} \min_{\Theta_k} \pi^n(\bar{\Theta}_k|\Theta_k) > 0 \end{aligned} \tag{16}$$

Due to the asymptotical convergence property of the proposed Markov chain, we have

$$\pi(\bar{\Theta}_k) = \lim_{n\to\infty} \max_{\Theta_k} \pi^n(\bar{\Theta}_k|\Theta_k) = \lim_{n\to\infty} \min_{\Theta_k} \pi^n(\bar{\Theta}_k|\Theta_k) \tag{17}$$

Since $\pi(\bar{\Theta}_k)$ lies between the minimum and maximum $\pi^n(\bar{\Theta}_k|\Theta_k)$ for any given state θ, (12) can be directly ensured. QED. □

Remark 2. Theorem 1 shows that, given any feasible joint association event as the initial state $\Theta_k^{(1)}$, the generated Markov chain $\left\{\Theta_k^{(t)}\right\}_{t=1}^{\infty}$ exponentially converges to the posterior of the joint event. Due to the convergence property, one can easily select a feasible joint event as the initial state for Gibbs sampler. For example, one can choose the joint association event that all targets are assumed to be miss-detected as the initial state of the Gibbs sampler, e.g., $\Theta_k^{(1)} = 0_{N_k}$.

Remark 3. Since Gibbs sampler is initialised with random values, samples generated at early iterations, known as the burn-in phase, usually cannot represent the target distribution and need to be discarded. Typically, there is no rule-of-thumb or analytically way to set the number of burn-in phase samples. However, due to the exponential convergence rate, the burn-in phase of the proposed Gibbs sampler is short. Even though the number of burn-in samples is empirically set, its influence on the marginalisation is ignorable since the generated Gibbs samples are not used for inference the stationary distribution (3). This will be empirically analysed in the simulation part.

Remark 4. Similar to Metropolis-Hastings sampling, the Gibbs sampling might become inefficient in exploring the space with high dimensionality, e.g., extremely large number of targets, because of the random-walk behaviour [49]. Under this condition, the Hamiltonian Monte Carlo sampling could be utilised as an alternative way to obtain the samples in a more efficient way. Compared to Gibbs sampling, the Hamiltonian Monte Carlo sampling has faster convergence speed for high-dimensional target distribution, although the price of single iteration is higher [49].

The Gibbs sampling-based marginalisation algorithm developed is summarised in Algorithm 1.

Algorithm 1 Marginalisation by Gibbs sampling.

Input: Previous target estimation, received measurements, allowable samples N_{gibbs}, burn-in samples $N_{burn-in}$
Output: Marginal association probability β_j^i

```
iter ← 1 {set the initial iteration counter as one}
N_{θ_j^i} ← 0 {set the event counter as zero}
Θ_k^{(1)} ← 0_{N_k} { set the initial state for the Gibbs sampler }
while iter < N_gibbs do
    iter ← iter + 1
    for m = 1 : N_k do
        Recursive sampling according to (11)
    end for
    Θ_k^{(iter)} = (θ_k^1, ..., θ_k^{N_k}) {one Gibbs sample}
    if iter > N_{burn-in} then
        if θ_k^i = j then
            N_{θ_j^i} ← N_{θ_j^i} + 1
        end if
    end if
end while
β_j^i = N_{θ_j^i} / (N_gibbs − N_{burn-in})
```

4. Distributed UKF-based information JPDA filter

This section proposes a distributed UKF-based information JPDA filter. We first briefly review the well-known average consensus algorithm and then present the detailed filtering algorithm. As this section only considers distributed estimation for each manoeuvre mode, we ignore the mode notation here for simplicity.

4.1. Average consensus

Suppose that N_s sensors participate in a cooperative distributed estimation mission. For this multi-sensor system, we use an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ to represent the communication topology, where $\mathcal{V} = \left\{v_1, v_2, \ldots, v_{N_s}\right\}$ is a set of vertices that represent N_s sensors and $\mathcal{E} = \{(i,j) \in \mathcal{V} \times \mathcal{V}\}$ is a set of edges that stand for the relationship between two neighbouring sensors in this topology. If two sensors (i and j) are adjacent, namely, they can communicate with each other, then $(v_i, v_j) \in \mathcal{E}$ and $(v_i, v_j) \in \mathcal{E}$. The graph $\mathcal{G}$ is said to be connected if there exists a path between any two sensors. The adjacency matrix of graph $\mathcal{G}$, denoted by $A = [a_{ij}] \in \mathbb{R}^{N_s \times N_s}$ is defined as $a_{ij} = 1$, if $(v_i, v_j) \in \mathcal{E}$, otherwise $a_{ij} = 0$.

To perform estimation fusion in a distributed way, the concept of consensus is adopted here. In particular, average consensus among networked sensors is performed. The average consensus algorithm is used to obtain the mean value of the information of all sensors in a distributed way. Denote a_l as the available information from the lth sensor and a_l is initialised as $a_l(0)$. Then, the distributed average consensus algorithm [18,19] at the mth iteration is defined as

$$a_{l,m} = a_{l,m-1} + \varepsilon \sum_{(v_l, v_{l'}) \in \mathcal{E}_l} \left(a_{l',m-1} - a_{l,m-1}\right) \tag{18}$$

where $\mathcal{E}_l$ denotes the set of sensors that have connections with the lth sensor and ε is the consensus gain, which is designed to tune the convergence speed. To guarantee the stability of the consensus phase, the gain ε should satisfy $\varepsilon \in (0, 1/\Delta_{\max})$, where $\Delta_{\max}$ is the maximum degree of undirected graph $\mathcal{G}$.

Based on the analysis of [18,19], it can be concluded that

$$\lim_{m\to\infty} a_{l,m} = \frac{1}{N_s} \sum_{l=1}^{N_s} a_l(0) \tag{19}$$

which means that the information of all sensors asymptotically converges to the average value.

4.2. UKF-based distributed estimation

This paper considers manoeuvring target tracking with a nonlinear observation model. In this regard, the well-known UKF is utilised to handle the issue of nonlinearity. It is known that the information filter is a suitable formula to address multi-sensor data fusion problem in a distributed manner[23,50]. However, direct application of the well-established information filter for single target tracking to the MTT is intractable due to the measurement origin uncertainty. To this end, this paper develops a distributed information-form of the JPDA filter that only exploits the exchanged information among the neighbour sensors. This form of the filter is excepted to enable low communication load, fast implementation and more robustness against sensor failures than the centralised implementation. Since the prediction step in multi-sensor filtering is similar to standard prediction of JPDA, we only derive the correction step in this subsection.

The state estimation in JPDA filter can be rewritten using matrix inversion lemma as

$$\begin{aligned}\hat{x}^i_{k|k} &= \hat{x}^i_{k|k-1} + \left[P^i_{k|k-1}\left(H^i_k\right)^T\left(S^i_k\right)^{-1}\right]\tilde{z}^i_k \\ &= \hat{x}^i_{k|k-1} + \left[\left(P^i_{k|k-1}\right)^{-1} + \left(H^i_k\right)^T R^{-1} H^i_k\right]^{-1} \\ &\quad \times \left(H^i_k\right)^T R^{-1}\left(\sum_{j=1}^{M_k}\beta^i_j z^i_{j,k} - \left(1-\beta^i_0\right)H^i_k\hat{x}^i_{k|k-1}\right) \\ &= \hat{x}^i_{k|k-1} + \left(Y^i_{k|k-1} + \mathrm{I}^i_k\right)^{-1}\left[\mathrm{i}^i_k - \left(1-\beta^i_0\right)\mathrm{I}^i_k\hat{x}^i_{k|k-1}\right] \\ &= \left(Y^i_{k|k-1} + \mathrm{I}^i_k\right)^{-1}\left(y^i_{k|k-1} + \mathrm{i}^i_k + \beta^i_0\mathrm{I}^i_k\hat{x}^i_{k|k-1}\right) \end{aligned} \tag{20}$$

where the information-related terms are defined as

$$\begin{aligned} Y^i_{k|k-1} &= \left(P^i_{k|k-1}\right)^{-1}, y^i_{k|k-1} = \left(P^i_{k|k-1}\right)^{-1}\hat{x}^i_{k|k-1} \\ \mathrm{I}^i_k &= \left(H^i_k\right)^T R^{-1} H^i_k, \quad \mathrm{i}^i_k = \left(H^i_k\right)^T R^{-1}\sum_{j=1}^{M_k}\beta^i_j z^i_{j,k} \end{aligned} \tag{21}$$

Define a new information contribution $\bar{\mathrm{i}}^i_k = \mathrm{i}^i_k + \beta^i_0\mathrm{I}^i_k\hat{x}^i_{k|k-1}$, then, (20) can be reduced to

$$\hat{x}^i_{k|k} = \left(Y^i_{k|k-1} + \mathrm{I}^i_k\right)^{-1}\left(y^i_{k|k-1} + \bar{\mathrm{i}}^i_k\right) \tag{22}$$

Based on the matrix inversion lemma, the correction of the information matrix $Y^i_{k|k-1}$ is derived as

$$\begin{aligned} Y^i_{k|k} &= \left\{P^i_{k|k-1} - K^i_k\left[\left(1-\beta^i_0\right)S^i_k - \bar{P}^i_k\right]\left(K^i_k\right)^T\right\}^{-1} \\ &= Y^i_{k|k-1} + \bar{\mathrm{I}}^i_k \end{aligned} \tag{23}$$

where $\bar{P}^i_k$ is a positive semi-definite matrix representing the measurement origin uncertainty and takes the form

$$\bar{P}^i_k = \sum_{j=1}^{M_k}\beta^i_j\tilde{z}^i_{j,k}\left(\tilde{z}^i_{j,k}\right)^T - \tilde{z}^i_k\left(\tilde{z}^i_k\right)^T \tag{24}$$

and

$$\bar{\mathrm{I}}^i_k = Y^i_{k|k-1}K^i_k\left\{\left[\left(1-\beta^i_0\right)S^i_k - \bar{P}^i_k\right]^{-1} - \left(K^i_k\right)^T Y^i_{k|k-1}K^i_k\right\}\left(K^i_k\right)^T Y^i_{k|k-1} \tag{25}$$

is an information matrix contribution.

Eqs. (22) and (23) constitute the information form of the JPDA filter. Different from classical information filter, the information form of JPDA filter consists of two different information state contributions and two different information matrix contributions. The differences between i^i_k and $\bar{\mathrm{i}}^i_k$, I^i_k and $\bar{\mathrm{I}}^i_k$ are resulted from the measurement origin uncertainty. Apparently, if there is no measurement uncertainty, we have $\beta^i_0 = 0$ and $\bar{P}^i_k = 0$, which means that the proposed information JPDA filter reduces to the classical information filter[19]. Furthermore, if the ith target is miss detected, then, $\beta^i_0 = 1$ and $\bar{P}^i_k = 0$, which implies that $\bar{\mathrm{i}}^i_k = \left(H^i_k\right)^T R^{-1}H^i_k\hat{x}^i_{k|k-1}$ and $Y^i_{k|k} = Y^i_{k|k-1}$. Therefore, if the ith target is miss detected by one sensor, that sensor can only provide the information about the prediction of the ith target.

Note that implementing (20) requires the measurement matrix H^i_k, which is not explicitly given by the nonlinear measurement model. In order to apply UKF in nonlinear filtering to the JPDA filter, we use the pseudo measurement matrix that can be derived from the statistical linear error propagation approach as $H^i_k \approx \left(P^i_{k|k-1}\right)^{-1}P^i_{k,xz}$, where $P^i_{k,xz}$ is the cross-correlation covariance, which can be approximated by unscented transformation as

$$P^i_{k,xz} = \sum_{s=0}^{2n}W_s\left(\lambda^{i,s}_{k|k-1} - \hat{x}^i_{k|k-1}\right)\left(\gamma^{i,s}_k - \hat{z}^i_k\right) \tag{26}$$

where $\lambda^{i,s}_{k|k-1}$ denotes the mapped sigma points through system transformation function, $\gamma^{i,s}_k$ the mapped sigma points through system observation function, W_s the weights of sigma points, and $\hat{z}^i_k$ the predicted measurement of the ith target, which is approximated by $\hat{z}^i_k = \sum_{s=0}^{2n}W_s\gamma^{i,s}_k$.

Based on the property of estimators with information form, incorporating additional information from other sensors could be achieved by summation of the corresponding information terms. This implies that the optimal centralised implementation of JPDA with N_s sensors is given by

$$\begin{aligned} Y^i_{k|k} &= Y^i_{k|k-1} + \sum_{l=1}^{N_s}\bar{\mathrm{I}}^i_{l,k} \\ \hat{x}^i_{k|k} &= \left(Y^i_{k|k-1} + \sum_{l=1}^{N_s}\mathrm{I}^i_{l,k}\right)^{-1}\left(y^i_{k|k-1} + \sum_{l=1}^{N_s}\bar{\mathrm{i}}^i_{l,k}\right) \end{aligned} \tag{27}$$

It follows from (27) that centralised estimation requires full information of all sensors. Considering each sensor usually can only communicate with its neighbours due to communication limit, this paper develops a distributed implementation based on consensus algorithm to recover the performance of the centralised estimation (27). Assume that the information states and matrices of all sensors converge to the global ones at previous scan, e.g., each sensor has an identical copy of the system state and the same amount of information matrix after consensus at previous scan, (27) can be reformulated as

$$\begin{aligned} Y^i_{k|k} &= \sum_{l=1}^{N_s}\left(\frac{Y^i_{l,k|k-1}}{N_s} + \bar{\mathrm{I}}^i_{l,k}\right) \\ \hat{x}^i_{k|k} &= \left[\sum_{l=1}^{N_s}\left(\frac{Y^i_{l,k|k-1}}{N_s} + I^i_{l,k}\right)\right]^{-1}\left(\sum_{l=1}^{N_s}\left(\frac{y^i_{l,k|k-1}}{N_s} + \bar{\mathrm{i}}^i_{l,k}\right)\right) \end{aligned} \tag{28}$$

Define consensus variables $v^i_{l,k}$, $V^i_{l,k}$, $G^i_{l,k}$, which are initialised as

$$\begin{aligned} v^i_{l,k}(0) &= \frac{y^i_{l,k|k-1}}{N_s} + \bar{\mathrm{i}}^i_{l,k}, \quad V^i_{l,k}(0) = \frac{Y^i_{l,k|k-1}}{N_s} + \bar{\mathrm{I}}^i_{l,k} \\ G^i_{l,k}(0) &= \frac{Y^i_{l,k|k-1}}{N_s} + \mathrm{I}^i_{l,k} \end{aligned} \tag{29}$$

In a practical implementation scenario of multi-sensor estimation, not all sensors can get the measurement information of each target due to limited sensor field-of-view and non-unity detection probability. In the case where no measurement information of the ith target is available at the lth sensor, the quality of local estimation $\hat{x}^i_{k|k}$ will be very poor and is far from the real state $x^i_{k|k}$. Fusing this poor information with other relatively good local estimation will obviously deteriorate the performance of the fused results and might result in estimation divergence. To accommodate this issue, we set the initial values of three consensus variables as zero if the local sensor node cannot get the measurement of a specific target. In this situation, the naive sensors will not perform information fusion steps and leverage the information from other non-naive sensors for estimation update. This simple strategy is demonstrated to be helpful in improving the stability of the fusion process.

After several average consensus iterations, each sensor obtains the distributed estimation of the system state and information matrix as

$$\begin{aligned} Y^i_{l,k|k} &= N_s V^i_{l,k} \\ \hat{x}^i_{l,k|k} &= \left(N_s G^i_{l,k}\right)^{-1}\left(N_s v^i_{l,k}\right) = \left(G^i_{l,k}\right)^{-1}v^i_{l,k} \end{aligned} \tag{30}$$

Based on the above derivations, the following points of the proposed distributed information JPDA filter are important.

(1) The consensus variables $v^i_{l,k}$ and $V^i_{l,k}$ contain the effect of measurement origin uncertainty. This shows how data association is tightly integrated in consensus-based distributed filtering, which has not been explored in previous works.

(2) It follows from (30) that the proposed distributed multi-target tracking algorithm requires the total number of sensors N_s for implementation. This information can be calculated in a distributed way as shown in [51]. In the case of sensor failure, however, one may get the wrong estimation of N_s. In Sec. VI, we will show that, even under the

condition of sensor failure, the proposed algorithm can get comparable performance to the centralised one.

(3) Theoretically, the average consensus guarantees asymptotic stability. In practice, since only finite number of iterations is tractable, convergence will not be fully achieved. However, we can safely assume that both the estimated system state and the information matrix of all sensors are the equal after enough finite iterations. This assumption is a key point that we can apply average consensus algorithm in distributed filtering and is reasonable as the consensus error could be made arbitrarily small after sufficient iterations.

Remark 5. In MTT over a sensor network, each sensor node orders its estimated tracks differently and therefore track-to-track association is required to associate the tracks from different sensors that represent the ith target. Typical track-to-track association utilises the so-called multi-dimensional association (MDA) formulation. If only two sensors are utilised in fusion, the MDA problem reduces to a classical 2D assignment problem, which can be efficiently solved by the well-known Hungarian algorithm. When the number of sensor nodes is larger than two, the track-to-track association MDA problem becomes NP-hard. There are a number of elegant choices for solving the MDA problem in combinatorial optimisation by reformulating the problem as a network flow [52] or using approximate Lagrangian relaxation [53,54] and stochastic sampling approach [55]. However, the discussion of track-to-track association is beyond the scope of this paper and we assume local estimates have perfect matching when evaluating the performance of the proposed algorithm for simplicity.

5. Distributed UKF-based multiple model information JPDA filter

This section develops the distributed UKF-based multiple model information JPDA filter to provide the capability for accurately estimating manoeuvring targets based on the JMNS modelling using IMM concept [42,56].

Let Z_k^l denote the measurement set received from the lth sensor at scan k and define $Z_k^{1:N_s} = \{Z_k^1, \dots, Z_k^{N_s}\}$. In general, one filtering cycle of JMNS consists of four steps:

(1) $p\left(x_{k-1}^i \middle| r_{k-1}^i, Z_{k-1}^{1:N_s}\right) \overset{\text{Mixing}}{\to} p\left(x_{k-1}^i \middle| r_k^i, Z_{k-1}^{1:N_s}\right)$

(2) $p\left(x_{k-1}^i \middle| r_k^i, Z_{k-1}^{1:N_s}\right) \overset{\text{Prediction}}{\to} p\left(x_k^i \middle| r_k^i, Z_{k-1}^{1:N_s}\right)$

(3) $p\left(x_k^i \middle| r_k^i, Z_{k-1}^{1:N_s}\right) \overset{\text{Bayes}}{\to} p\left(x_k^i \middle| r_k^i, Z_k^{1:N_s}\right)$

(4) $p\left(r_k^i \middle| Z_{k-1}^{1:N_s}\right) \overset{\text{Bayes}}{\to} p\left(r_k^i \middle| Z_k^{1:N_s}\right)$

Since steps (2) and (3) are accomplished by the proposed distributed information JPDA filter, this section only focuses on steps (1) and (4). According to the total probability theorem, the mixed prior can be derived as

$$\begin{aligned} p\left(x_{k-1}^i \middle| r_k^i = r, Z_{k-1}^{1:N_s}\right) = &\sum_{m=1}^{\mathcal{M}_r} p\left(x_{k-1}^i \middle| r_{k-1}^i = m, Z_{k-1}^{1:N_s}\right) \\ &\times \Pr\left(r_{k-1}^i = m \middle| r_k^i = r, Z_{k-1}^{1:N_s}\right) \end{aligned} \tag{31}$$

and the mode mix probability $\Pr\left(r_{k-1}^i = m \middle| r_k^i = r, Z_{k-1}^{1:N_s}\right)$ is given by

$$\Pr\left(r_{k-1}^i = m \middle| r_k^i = r, Z_{k-1}^{1:N_s}\right) = \frac{\pi_{mr} p\left(r_{k-1}^i = m \middle| Z_{k-1}^{1:N_s}\right)}{\sum_{m=1}^{\mathcal{M}_r} \pi_{mr} p\left(r_{k-1}^i = m \middle| Z_{k-1}^{1:N_s}\right)} \tag{32}$$

It follows from (31) that the exact solution of JMNS estimation is a Gaussian sum with $(\mathcal{M}_r)^k$ terms at scan k. Propagation of Gaussian mixtures is naturally intractable in real applications. In order to maintain the computational efficiency, the concept of IMM is adopted here by using a single Gaussian to approximate the mixed prior (31) at every time instant. This implies that the mixed initial condition for each filter is given by

$$\begin{aligned} \hat{x}_{k-1|k-1}^{0i,r} &= \sum_{m=1}^{\mathcal{M}_r} \mu_{k-1}^{i,r|m} \hat{x}_{k-1|k-1}^{i,m} \\ P_{k-1|k-1}^{0i,r} &= \sum_{m=1}^{\mathcal{M}_r} \mu_{k-1}^{i,r|m} \left[P_{k-1|k-1}^{i,m} + \left(\hat{x}_{k-1|k-1}^{i,m} - \hat{x}_{k-1|k-1}^{0i,r}\right) \right. \\ &\quad \left. \times \left(\hat{x}_{k-1|k-1}^{i,m} - \hat{x}_{k-1|k-1}^{0i,r}\right)^T \right] \end{aligned} \tag{33}$$

where $\mu_{k-1}^{i,r|m} = \Pr\left(r_{k-1}^i = m \middle| r_k^i = r, Z_{k-1}^{1:N_s}\right)$.

By feeding the mixed prior $\left(\hat{x}_{k-1|k-1}^{0i,r}, P_{k-1|k-1}^{0i,r}\right)$ as the initial condition to each information JPDA filter, only $\mathcal{M}_r$ modes are kept at one estimation cycle.

Based on the mode-conditioned update $p\left(x_k^i \middle| r_k^i, Z_k^{1:N_s}\right)$ and the mode probability update $p\left(r_k^i \middle| Z_k^{1:N_s}\right)$, the posterior probability density function of one target can be represented by a Gaussian mixture distribution using the total probability theorem as

$$\begin{aligned} p\left(x_k^i \middle| Z_k^{1:N_s}\right) = &\sum_{r=1}^{\mathcal{M}_r} p\left(x_k^i \middle| r_k^i = r, Z_k^{1:N_s}\right) \\ &\times p\left(r_k^i = r \middle| Z_k^{1:N_s}\right) \end{aligned} \tag{34}$$

The state estimate and error covariance matrix of each target, extracted from (34) using moment-matching, are obtained as

$$\begin{aligned} \hat{x}_{k|k}^i &= \sum_{r=1}^{\mathcal{M}_r} \mu_{k|k}^{i,r} \hat{x}_{k|k}^{i,r} \\ P_{k|k}^i &= \sum_{r=1}^{\mathcal{M}_r} \mu_{k|k}^{i,r} \left[P_{k|k}^{i,r} + \left(\hat{x}_{k|k}^{i,r} - \hat{x}_{k|k}^i\right)\left(\hat{x}_{k|k}^{i,r} - \hat{x}_{k|k}^i\right)^T \right] \end{aligned} \tag{35}$$

where $\mu_{k|k}^{i,r} = p\left(r_k^i = r \middle| Z_k^{1:N_s}\right)$ is the posterior mode probability, which can be calculated by Bayesian rule as

$$\mu_{k|k}^{i,r} = \frac{p\left(r_k^i = r \middle| Z_{k-1}^{1:N_s}\right)}{p\left(Z_k^{1:N_s} \middle| Z_{k-1}^{1:N_s}\right)} p\left(Z_k^{1:N_s} \middle| r_k^i = r, Z_{k-1}^{1:N_s}\right) \tag{36}$$

where $p\left(Z_k^{1:N_s} \middle| Z_{k-1}^{1:N_s}\right)$ is the normalisation constant, and the predicted mode probability $p\left(r_k^i = r \middle| Z_{k-1}^{1:N_s}\right)$ can be obtained as

$$p\left(r_k^i = r \middle| Z_{k-1}^{1:N_s}\right) = \sum_{m=1}^{\mathcal{M}_r} \pi_{mr} p\left(r_{k-1}^i = m \middle| Z_{k-1}^{1:N_s}\right) \tag{37}$$

Since the measurements from different sensors are independent, (36) can be further reduced to

$$\mu_{k|k}^{i,r} = \frac{p\left(r_k^i = r \middle| Z_{k-1}^{1:N_s}\right)}{p\left(Z_k^{1:N_s} \middle| Z_{k-1}^{1:N_s}\right)} \prod_{l=1}^{N_s} p\left(Z_k^l \middle| r_k^i = r, Z_{k-1}^{1:N_s}\right) \tag{38}$$

Eq. (38) provides the centralised form of mode probability calculation and is required to be distributed for sensor networks. Since the product form in (38) causes an issue in directly applying the average consensus algorithm in the proposed distributed fusion, we define an auxiliary variable $\delta_{l,k}^{i,r} = \ln \Lambda_{l,k}^{i,r}$. Note that $\Lambda_{l,k}^{i,r} = p\left(Z_k^l \middle| r_k^i = r, Z_{k-1}^{1:N_s}\right)$ is the mode conditioned measurement likelihood of the lth sensor. Then, we define consensus variable $Q_{l,k}^{i,r}$, which is initialised as $Q_{l,k}^{i,r}(0) = \delta_{l,k}^{i,r}$. After running average consensus algorithm for $Q_{l,k}^{i,r}$, the fusion of the posterior mode probability in a distributed way is given by

$$\mu_{k|k}^{i,r} = \frac{p\left(r_k^i = r \middle| Z_{k-1}^{1:N_s}\right)}{p\left(Z_k^{1:N_s} \middle| Z_{k-1}^{1:N_s}\right)} \exp\left(N_s\left(\frac{1}{N_s}\sum_{l=1}^{N_s}\delta_{l,k}^{i,r}\right)\right) = \frac{p\left(r_k^i = r \middle| Z_{k-1}^{1:N_s}\right)}{p\left(Z_k^{1:N_s} \middle| Z_{k-1}^{1:N_s}\right)} \exp\left(N_s Q_{l,k}^{i,r}\right) \tag{39}$$

After finding the posterior mode probability $\mu_{k|k}^{i,r}$, the final state estimate and covariance matrix of each target is given by (35).

Remark 6. Consensus on $Q_{l,k}^{i,r}$ requires $\Lambda_{l,k}^{i,r}$: this can be readily obtained by summing up all the PDA probability (4), i.e.,

$$\Lambda_{l,k}^{i,r} = \left(1 - P_D P_G\right)\lambda_F + \sum_{j\neq 0}^{M_{l,k}^{i,r}} p\left(z_{l,j,k} \middle| x_{l,k}^{i,r}\right) P_D \tag{40}$$

where $M_{l,k}^{i,r}$ denotes the number of validated measurements for the ith target with the rth mode from the lth sensor.

Remark 7. Compared with single-mode distributed JPDA filter, incorporating IMM with mode probability fusion will inevitably increase the computational burden. However, it is clear to verify that the complexity of the proposed distributed UKF-based multiple model information JPDA filter is proportional to the number of modes $\mathcal{M}_r$, e.g., the proposed algorithm is scalable.

The proposed multiple model UKF-based distributed information JPDA filter is summarised in Algorithm 2.

6. Numerical simulations

In this section, the effectiveness of the proposed multiple model UKF-based distributed information JPDA filtering algorithm is demonstrated through numerical simulations in a cluttered environment. The target tracking problem is based on some generic air traffic control (ATC) scenarios. For typical ATC cases, the behaviour of civilian aircraft may be modelled by two different modes: constant velocity (CV) and coordinated turning (CT).

6.1. Simulation setup

In our simulations, 7 targets, switching between CV model and CT model, are considered. The state vector contains planar position and velocity. More specifically, for CV model, the state transition is

$$x_k = F_{CV} x_{k-1} + G w_{k-1} \tag{41}$$

with

$$F_{CV} \triangleq \mathbb{I}_{2\times 2} \otimes \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad G \triangleq \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix} \tag{42}$$

where $\mathbb{I}_{2\times 2}$ denotes the 2×2 identity matrix, $T = 1s$ the sampling period, and $w_k \sim \mathcal{N}\left(\cdot; 0, \sigma_v^2\right)$ the Gaussian process noise with $\sigma_v = 5m/s^2$. The state transition of CT model is

$$x_k = F_{CT} x_{k-1} + G w_{k-1} \tag{43}$$

with

$$F_{CT} \triangleq \begin{bmatrix} 1 & \frac{\sin(\omega_k T)}{\omega_k} & 0 & -\frac{1-\cos(\omega_k T)}{\omega_k} \\ 0 & \cos(\omega_k T) & 0 & -\sin(\omega_k T) \\ 0 & \frac{1-\cos(\omega_k T)}{\omega_k} & 1 & \frac{\sin(\omega_k T)}{\omega_k} \\ 0 & \sin(\omega_k T) & 0 & \cos(\omega_k T) \end{bmatrix} \tag{44}$$

$$G \triangleq \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix} \tag{44}$$

Algorithm 2 Distributed UKF-based multiple model information JPDA filter.

Input: Previous target estimation, received measurements
Output: Current target estimation
(1) **Step 1. Mode Mixing**
a. Calculate the mode mix probability $\mu_{k-1}^{i,r|m}$ using (32)
b. Calculate the mixed prior state estimation $\hat{x}_{k-1|k-1}^{0i,r}$ and error covariance $P_{k-1|k-1}^{0i,r}$ using (33)
(2) **Step 2. Mode-conditioned distributed JPDA estimation** (Sec. 4)
a. Predict target state and calculate the error covariance based on $\hat{x}_{l,k-1|k-1}^{0i,r}$ and $P_{l,k-1|k-1}^{0i,r}$
b. Receive measurements and perform gating with probability P_G
c. Apply Gibbs sampling for marginal association probability β_j^i approximation (Sec. 3)
d. Calculate the information terms $Y_{l,k|k-1}^i$, $y_{l,k|k-1}^i$, $\mathrm{i}_{l,k}^i$, $\mathrm{I}_{l,k}^i$, $\bar{\mathrm{i}}_{l,k}^i$, $\bar{\mathrm{I}}_{l,k}^i$
e. Broadcast message to neighbour sensors and receive neighbours' messages on $v_{l,k}^{i,r}$, $V_{l,k}^{i,r}$, $G_{l,k}^{i,r}$
f. Perform average consensus for each $v_{l,k}^{i,r}$, $V_{l,k}^{i,r}$, $G_{l,k}^{i,r}$ independently
g. Get fused posterior target estimation $\hat{x}_{k|k}^{i,r}$ and $P_{k|k}^{i,r}$ for each mode
(3) **Step 3. Mode probability fusion and update** (Sec.5)
a. Calculate the mode conditioned measurement likelihood (40)
b. Broadcast message to neighbour sensors and receive neighbours' messages on $\delta_{l,k}^{i,r}$
c. Perform average consensus for $\delta_{l,k}^{i,r}$
d. Calculate fused posterior mode probability $\mu_{k|k}^{i,r}$ according to (39)
(4) **Step 4. Output**
Given all the mode-conditioned fused estimates, the final state estimate of each target is obtained as a weighted sum of individual fused estimates by using (35)

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where $\omega_k = 6\pi/180 rad/s$ is the turning rate. The nonlinear range and bearing measurement model for state correction is

$$z = \begin{bmatrix} \sqrt{\left(x_T - x_R\right)^2 + \left(y_T - y_R\right)^2} \\ \arctan\left(\frac{y_T - y_R}{x_T - x_R}\right) \end{bmatrix} + v_k \tag{45}$$

where (x_T, y_T) is target position, (x_R, y_R) radar position, and $v_k \sim \mathcal{N}\left(\cdot; 0, R_k\right)$ the Gaussian measurement noise with $R_k = diag\left(\sigma_r^2, \sigma_a^2\right)$, $\sigma_r = 20m$, $\sigma_a = 2(\pi/180) rad$.

For all sensors, the measurements are generated with a detection probability $P_D = 0.85$ and the clutter is assumed to be uniformly distributed in the surveillance region with its number being Poisson with 10 average returns at each scan. Gating is performed with a threshold such that the gating probability is $P_G = 0.999$. The field-of-view of all radars are set as $[0, 3000m] \times [0, 90°]$. In order to fully cover the entire surveillance region, we use the minimum four radars, which are fixed at $(-1500m, -600m)$, $(-1500m, 2000m)$, $(1600m, -600m)$, $(1600m, 2000m)$. The default communication topology among these four radars is represented by the adjacent matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \tag{46}$$

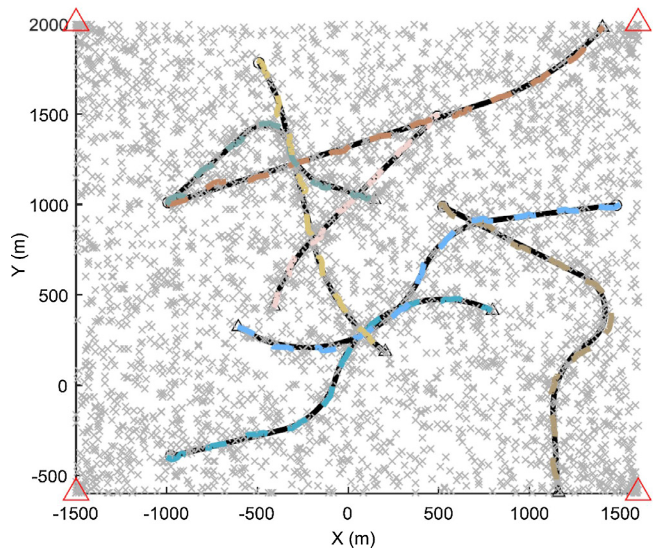


Fig. 2. Snapshots of the considered scenario with grey stars as measurements, black solid line ground truth, colour dashed line estimation, red triangle sensors. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

It follows from (46) that the maximum degree of the network undirected graph is $\Delta_{\max} = 2$ and then the consensus gain is set as $\varepsilon = 0.5/2$. For information exchange, the consensus iteration is selected as $Iter_{\max} = 10$, which is shown to be enough to reduce the consensus error. The design parameters for Gibbs sampling-based implementation are set as $n_{\max} = 200$, $n_{burn-in} = 100$. All experiments are performed on Matlab 2016b platform using an Intel Core i5-6500 CPU and the number of Monte-Carlo runs is 50.

In simulations, three different models are considered, i.e. CV model, left CT model, right CT model. The mode transformation probability is selected as

$$\Pi = \left\{\pi_{mr}\right\}_{3\times 3} = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \tag{47}$$

Snapshots of the considered scenario are depicted in Fig. 2. The well-known unitless optimal sub-pattern assignment (OSPA) distance metric [57] for MTT problem is considered here for performance evaluation. Let X and Y be the position estimation set and true target position set, respectively. The cardinality of these two sets are m and n, respectively. Denote Π_n as the set of all permutations on $\{1, 2, \dots, n\}$ for any positive integer n. $d^c\left(x_i, y_{\pi(i)}\right) = \min\left(d\left(x_i, y_{\pi(i)}\right), c\right)$ with $d(x_i, y_{\pi(i)})$ is the cut-off Euclidean distance between two vectors with $d(x_i, y_{\pi(i)})$ being the Euclidean distance. Then, for $c > 0$ and $1 \le p < \infty$, the OSPA distance $d_p^c(X, Y)$ is defined as [57]

$$d_p^c(X, Y) \triangleq \begin{cases} \left[\frac{1}{n}\left(\min_{\pi\in\Pi_n} \sum_{i=1}^{m} d^c\left(x_i, y_{\pi(i)}\right)^p + c^p(n-m)\right)\right]^{1/p}, & m \le n \\ d_p^c(Y, X), & m > n \end{cases} \tag{48}$$

where the order parameter p determines the sensitivity of $d_p^c(X, Y)$ in penalizing estimation outliers, while the cut-off parameter c determines the relative weighting of the penalties allocated to cardinality and localization errors. In all simulations, these two parameters are set as $p = 2$, $c = 100$.

6.2. Characteristics of the proposed algorithm

For consensus-based filtering, multiple communications between different sensors are required and the performance is related to the consensus gain and the number of iterations. In order to investigate the effect of these two parameters on filtering performance, Monte-Carlo simulations are performed with respect to different iterations and consensus gain. The simulation results of the OSPA distance are depicted in Figs. 3 (a) and (b). When testing the effect of one parameter, the other one is set to its corresponding default value presented in Sec. VI A. Fig. 3 (a) shows that the performance of distributed estimation is improved when the number of iterations increase. Distributed JPDA filtering with iterations greater than 5 has close performance with its centralised JPDA counterpart. This implies that the performance of centralised estimation can be recovered with enough consensus iterations. Fig. 3 (b) shows that improvement in estimation can be obtained by increasing the consensus gain. However, with large enough gains, there is not much difference for the proposed distributed JPDA with different consensus gains.

As mentioned before, the number of the burn-in samples is empirically set. Fig. 3 (c) studies the impact of number burn-in samples on the performance of the proposed tracking algorithm. From this figure, it is clear that $n_{burn-in} = 60$ achieves the best estimation accuracy, but the tracking performance in terms of the OSPA distance under all considered conditions is comparable. This means that the proposed algorithm is robust against the variation of number of burn-in samples.

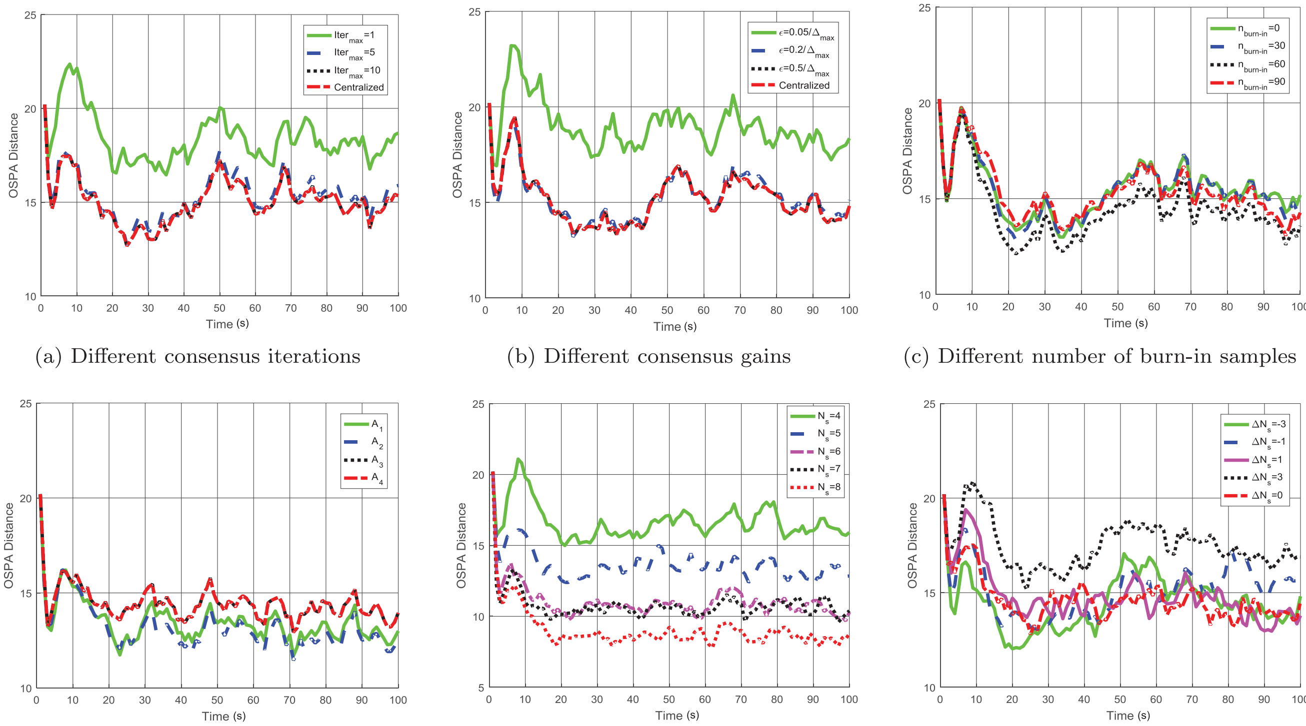
In distributed network estimation, the communication structure plays an important role in governing the overall filtering performance. The performance of the proposed algorithm is compared by using four different sensor networks A_1, A_2, A_3 and A_4:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \tag{49}$$

The results, shown in Fig. 3 (d), reveal that the OSPA discrepancies between different communication topologies are quite small, demonstrating that proposed algorithm is robust against the variation of communication structures.

Now, let us investigate the effect of the total number of sensors on filtering performance. Except for the four default sensors mentioned in Fig. 2, the positions of other sensors are randomly placed. Fig. 3 (e) provides the OSPA distance of the Monte-Carlo simulations with respect to different total numbers of sensors. In this simulation, the sensor range is set as 3000m. One can note from Fig. 3 (e) that larger number of sensors renders smaller OSPA distance, but the difference in terms of estimation accuracy is ignorable when the sensor number is larger than 6. This result reveals that the minimum number of sensors that cover the entire surveillance region is enough for the proposed filter to obtain good performance. The recorded mean running time of these four scenarios are 14.9137s, 17.6921s, 20.3121s, 22.9994s, 25.0001s, respectively, which reveals that the running time of the proposed algorithm grows linearly with the increasing of sensor numbers.

As mentioned earlier, the proposed distributed multi-target tracking algorithm requires the total number of sensors for implementation. Although this information can be calculated in a distributed way as shown in [51], one may get the wrong estimation of N_s in the presence of sensor failure. To this end, we test the robustness and sensitivity of the proposed distributed filter against the wrong estimation of N_s. In this regard, let the total number of sensors be $N_s + \Delta N_s$ with ΔN_s being the biased term. Fig. 3(f) shows the OSPA distance of the Monte-Carlo simulations with respect to different biased terms. The actual sensor number for this test is 7. Fig. 3(f) reveals that the OSPA discrepancies between different biased terms are quite small, demonstrating that proposed algorithm is highly tolerant to the wrong estimation of N_s.



(a) Different consensus iterations (b) Different consensus gains (c) Different number of burn-in samples

(d) Different communication topologies (e) Different number of sensors (f) Different biased terms

Fig. 3. Characteristics of the proposed algorithm: OSPA distance under different conditions.

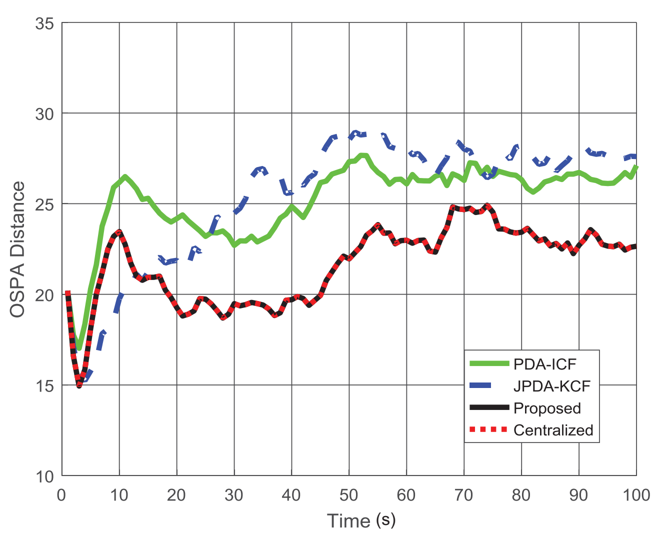


Fig. 4. OSPA distance of different algorithms.

6.3. Comparisons with other algorithms

In order to verify the effectiveness of the proposed algorithm compared to other PDA-type multi-sensor filters, we compare the proposed one with JPDA-KCF [25,26] and PDA-ICF [31]. To make fair comparisons, all tested algorithms are enhanced by the IMM for manoeuvring target tracking. Fig. 4 shows the OSPA distance of the Monte-Carlo simulations obtained by different algorithms. As not all sensors can the measurement of each target due to sensor field-of-view limit and none-unity detection probability in the considered scenario, there exist naive sensors for some targets at some time instants. This fact results in performance degradation of JPDA-KCF. Not surprisingly, since PDA only considers one measurement-target associations and neglect the effect of other possible solutions, not realistic for some scenarios, the obtained mean OSPA distance of the PDA-ICF is larger than that of the proposed algorithm.

To demonstrate the efficiency of Gibbs sampling-aided implementation, Monte-Carlo simulations are carried out and the comparison results among exact JPDA, ENNJPDA [12], m-best JPDA [15] and Gibbs sampling-aided implementation are presented in Figs. 5 and 6, where Fig. 5 is for different number of clutter returns at one scan and Fig. 6 is for different number of targets. In ENNJPDA, only the joint event with the highest probability is picked up for marginalisation. As an extension of this idea, m-best JPDA maintains m-best joint events for the marginalisation. In this regard, the ENNJPDA can be viewed as a special case of m-best JPDA with $m = 1$. In [15], it was shown that the m-best joint events can be iteratively solved by linear programming (LP). In the simulations, we leverage the commercial *Gurobi* solver to derive both ENNJPDA and m-best JPDA with $m = 5$. Additionally, in order to reduce the complexity of LP problems for m-best JPDA, the binary tree partition method [15] is also adopted in simulations for m-best JPDA implementation. The results in Figs. 5 (a) and 6 (a) reveal that the proposed Gibbs sampling-aided implementation runs much faster than standard JPDA as well as m-best JPDA. However, as ENNJPDA only needs to solve one LP problem to obtain the best joint event, it requires slightly less running time than the proposed method in a dense clutter environment or with large number of targets. Notably, the zoomed-in graph in Fig. 6 (a) demonstrates that the execution time of the proposed Gibbs sampling-aided JPDA grows linearly with respect to the target numbers and thereby the proposed algorithm is scalable. For large-scale problem, the scenario with 14 targets for instance, exact JPDA, m-best JPDA take 1374.8034s, 41.8488s, respectively, while Gibbs sampling-aided implementation only requires 25.9695s. It is evident from Figs. 5 (b) and 6 (b) that the estimation performance of exact JPDA, m-best JPDA and the proposed one in terms of OSPA distance accuracy is comparable. Fig. 5 (b) also shows that the mean OSPA distance of standard JPDA is relatively lower than the proposed algorithm and m-best JPDA when the number of average clutter returns increases. The reason of this phenomenon lies in that both

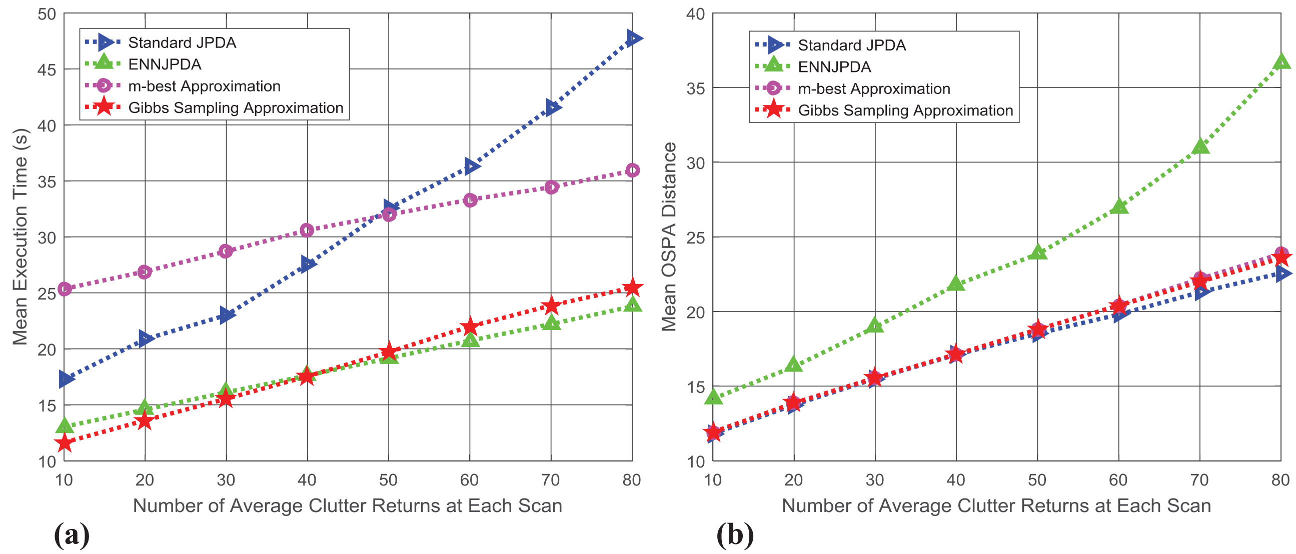


Fig. 5. Comparison results with different number of average clutter returns at each scan: (a) Mean execution time; and (b) Mean OSPA distance.

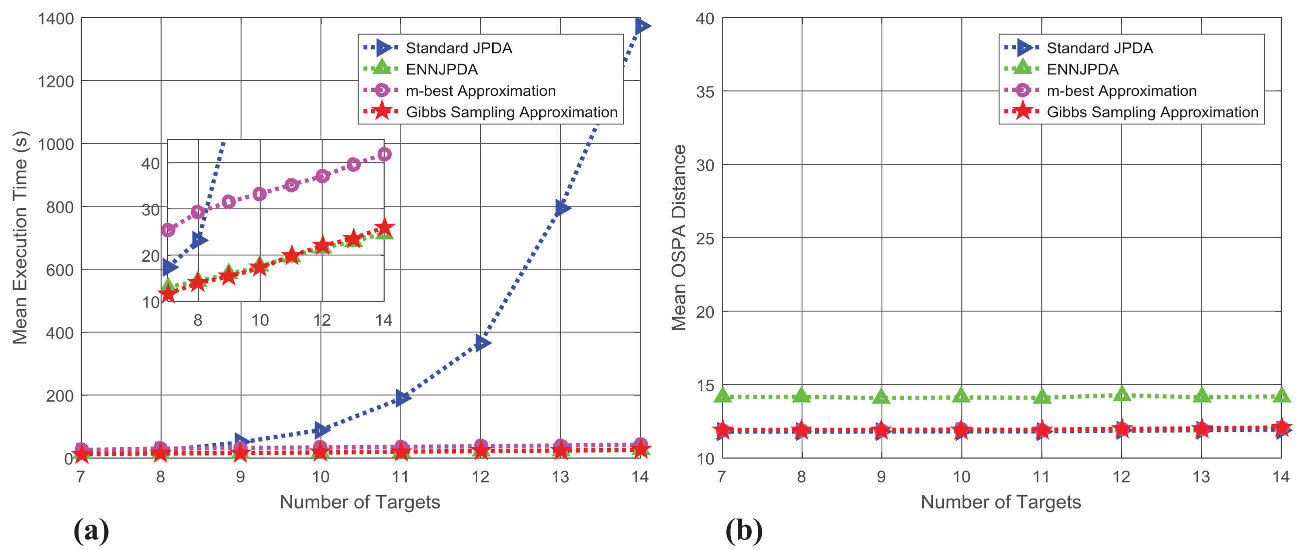


Fig. 6. Comparison results with different number of targets: (a) Mean execution time; and (b) Mean OSPA distance.

the proposed method and m-best JPDA use accountable approximations, which means that the number of Gibbs samples plays a trade-off role in real implementation. However, the recorded discrepancy of mean OSPA distance between these three schemes is less than 1.5, while ENNJPDA is highly sensitive to clutters. These results prove that the proposed Gibbs sampling method can greatly improve the efficiency of JPDA without sacrificing tracking performance.

7. Conclusions

The problem of multiple manoeuvring targets tracking using multiple sensors in a distributed fashion is investigated in this paper. The contribution of this work is twofold. As our first contribution, we propose a general framework of multi-target multi-sensor estimation for JMNSs. The proposed algorithm is derived by JPDA and consensus-based data fusion. Due to the combinatorial nature of JPDA, exact solution is computationally intractable. Our second contribution lies in developing an efficient Gibbs sampling method to approximate the marginal association probabilities in JPDA. Simulation results show that the proposed algorithm can generate almost the same estimation performance as the centralised filter and is much faster than classical JPDA without sacrificing tracking accuracy.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Lemmas 1 and 2

This appendix collects two key lemmas in [48] that are used in the proof of Theorem 1.

Lemma 1. *Let π be the transition matrix of a finite-state Markov chain and let π^n be the nth order transition probabilities. Then, for any state and $n \geq 1$, we have*

$$\max_{\theta} \pi^{n+1}(\chi|\theta) \leq \max_{\varsigma} \pi^{n}(\chi|\varsigma)$$
$$\min_{\theta} \pi^{n+1}(\chi|\theta) \geq \min_{\varsigma} \pi^{n}(\chi|\varsigma) \tag{A.1}$$

Lemma 2. *Let* $\alpha = \min_\theta \pi(\chi|\theta)$ *and the transition matrix* π *of a finite-state Markov chain satisfy* $\pi > 0$. *Then, for any state and* $n \geq 1$, *we have*

$$\begin{aligned}
&\max_\theta \pi^{n+1}(\chi|\theta) - \min_\theta \pi^{n+1}(\chi|\theta) \\
&\quad \leq \left[\max_\varsigma \pi^n(\chi|\varsigma) - \min_\varsigma \pi^n(\chi|\varsigma)\right](1-2\alpha) \\
&\max_\varsigma \pi^n(\chi|\varsigma) - \min_\varsigma \pi^n(\chi|\varsigma) \leq (1-2\alpha)^n \\
&\lim_{n\to\infty} \max_\varsigma \pi^n(\chi|\varsigma) = \lim_{n\to\infty} \min_\varsigma \pi^n(\chi|\varsigma) \geq \alpha > 0 \qquad \text{(A.2)}
\end{aligned}$$

CRediT authorship contribution statement

Shaoming He: Conceptualization, Methodology, Software, Visualization, Investigation, Validation, Writing - original draft. **Hyo-Sang Shin:** Conceptualization, Methodology, Investigation, Validation, Supervision, Writing - review & editing. **Antonios Tsourdos:** Supervision.

References

[1] S. He, H.-S. Shin, S. Xu, A. Tsourdos, Distributed estimation over a low-cost sensor network: a review of state-of-the-art, Inf. Fusion 54 (2020) 21–43.
[2] Y. Dan, J. Hongbing, G. Yongchan, A robust d–s fusion algorithm for multi-target multi-sensor with higher reliability, Inf. Fusion 47 (2019) 32–44.
[3] S. He, H.-S. Shin, A. Tsourdos, Distributed joint probabilistic data association filter with hybrid fusion strategy, IEEE Trans. Instrum. Meas. 69 (1) (2020) 286–300.
[4] J. Ding, S. Sun, J. Ma, N. Li, Fusion estimation for multi-sensor networked systems with packet loss compensation, Inf. Fusion 45 (2019) 138–149.
[5] D. Reid, An algorithm for tracking multiple targets, IEEE Trans. Automat. Contr. 24 (6) (1979) 843–854.
[6] S. Blackman, R. Popoli, Design and Analysis of Modern Tracking Systems, Artech House, Norwood, MA, USA, 1999.
[7] I.J. Cox, S.L. Hingorani, An efficient implementation of reid's multiple hypothesis tracking algorithm and its evaluation for the purpose of visual tracking, IEEE Trans. Pattern Anal. Mach. Intell. 18 (2) (1996) 138–150.
[8] A.B. Poore, A.J. Robertson III, A new lagrangian relaxation based algorithm for a class of multidimensional assignment problems, Comput. Optim. Appl. 8 (2) (1997) 129–150.
[9] J. Areta, Y. Bar-Shalom, M. Levedahl, K.R. Pattipati, Hierarchical track association and fusion for a networked surveillance system., J. Adv. Inf. Fusion 1 (2) (2006) 140–157.
[10] C. Kim, F. Li, A. Ciptadi, J.M. Rehg, Multiple hypothesis tracking revisited, in: Proceedings of the IEEE International Conference on Computer Vision, 2015, pp. 4696–4704.
[11] T. Fortmann, Y. Bar-Shalom, M. Scheffe, Sonar tracking of multiple targets using joint probabilistic data association, IEEE J. Oceanic Eng. 8 (3) (1983) 173–184.
[12] R.J. Fitzgerald, Development of practical pda logic for multitarget tracking by microprocessor, in: American Control Conference, IEEE, 1986, pp. 889–898.
[13] C. Morefield, Application of 0–1 integer programming to multitarget tracking problems, IEEE Trans. Automat. Contr. 22 (3) (1977) 302–312.
[14] J.A. Roecker, A class of near optimal jpda algorithms, IEEE Trans. Aerosp. Electron. Syst. 30 (2) (1994) 504–510.
[15] S. Hamid Rezatofighi, A. Milan, Z. Zhang, Q. Shi, A. Dick, I. Reid, Joint probabilistic data association revisited, in: Proceedings of the IEEE International Conference on Computer Vision, 2015, pp. 3047–3055.
[16] R. Carli, A. Chiuso, L. Schenato, S. Zampieri, Distributed kalman filtering based on consensus strategies, IEEE J. Sel. Areas Commun. 26 (4) (2008) 622–633.
[17] S. He, H.-S. Shin, A. Tsourdos, Multi-sensor multi-target tracking using domain knowledge and clustering, IEEE Sens. J. 18 (19) (2018) 8074–8084.
[18] R. Olfati-Saber, J.S. Shamma, Consensus filters for sensor networks and distributed sensor fusion, in: Proceedings of the 44th IEEE Conference on Decision and Control, IEEE, 2005, pp. 6698–6703.
[19] R. Olfati-Saber, Distributed kalman filtering for sensor networks, in: 2007 46th IEEE Conference on Decision and Control, IEEE, 2007, pp. 5492–5498.
[20] R. Olfati-Saber, J.A. Fax, R.M. Murray, Consensus and cooperation in networked multi-agent systems, Proc. IEEE 95 (1) (2007) 215–233.
[21] G. Battistelli, L. Chisci, G. Mugnai, A. Farina, A. Graziano, Consensus-based linear and nonlinear filtering, IEEE Trans. Automat. Contr. 60 (5) (2015) 1410–1415.
[22] A.T. Kamal, C. Ding, B. Song, J.A. Farrell, A.K. Roy-Chowdhury, A generalized kalman consensus filter for wide-area video networks, in: 50th IEEE Conference on Decision and Control and European Control Conference, IEEE, 2011, pp. 7863–7869.
[23] A.T. Kamal, J.A. Farrell, A.K. Roy-Chowdhury, Information weighted consensus, in: 51st IEEE Conference on Decision and Control (CDC), IEEE, 2012, pp. 2732–2737.
[24] A.T. Kamal, J.A. Farrell, A.K. Roy-Chowdhury, Information weighted consensus filters and their application in distributed camera networks, IEEE Trans. Automat. Contr. 58 (12) (2013) 3112–3125.
[25] N.F. Sandell, R. Olfati-Saber, Distributed data association for multi-target tracking in sensor networks, in: 47th IEEE Conference on Decision and Control, IEEE, 2008, pp. 1085–1090.
[26] A.K. Roy-Chowdhury, B. Song, Camera networks: the acquisition and analysis of videos over wide areas, Synthesis Lect. Comput. Vis. 3 (1) (2012) 1–133.
[27] W. Li, Y. Jia, Distributed estimation for markov jump systems via diffusion strategies, IEEE Trans. Aerosp. Electron. Syst. 53 (1) (2017) 448–460.
[28] G. Battistelli, L. Chisci, C. Fantacci, A. Farina, A. Graziano, Consensus-based multiple-model bayesian filtering for distributed tracking, IET Radar Sonar Navig. 9 (4) (2015) 401–410.
[29] C. Fantacci, G. Battistelli, L. Chisci, A. Farina, A. Graziano, Multiple-model algorithms for distributed tracking of a maneuvering target, in: Information Fusion (FUSION), 2012 15th International Conference on, IEEE, 2012, pp. 1028–1035.
[30] Y. Bar-Shalom, F. Daum, J. Huang, The probabilistic data association filter, IEEE Control Syst. Mag. 29 (6) (2009) 82–100.
[31] A.T. Kamal, J.A. Farrell, A.K. Roy-Chowdhury, Information consensus for distributed multi-target tracking, in: Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 2013, pp. 2403–2410.
[32] G. Battistelli, L. Chisci, C. Fantacci, A. Farina, A. Graziano, Consensus cphd filter for distributed multitarget tracking, IEEE J. Sel. Top. Signal Process. 7 (3) (2013) 508–520.
[33] M. Üney, D.E. Clark, S.J. Julier, Distributed fusion of phd filters via exponential mixture densities, IEEE J. Sel. Top. Signal Process. 7 (3) (2013) 521–531.
[34] T. Li, J.M. Corchado, S. Sun, Partial consensus and conservative fusion of gaussian mixtures for distributed phd fusion, IEEE Trans. Aerosp. Electron. Syst. 55 (5) (2018) 2150–2163.
[35] L. Gao, G. Battistelli, L. Chisci, Multiobject fusion with minimum information loss, arXiv preprint arXiv:1903.04239 (2019).
[36] T. Li, H. Fan, J. García, J.M. Corchado, Second-order statistics analysis and comparison between arithmetic and geometric average fusion: application to multi-sensor target tracking, Inf. Fusion 51 (2019) 233–243.
[37] C. Fantacci, B.-N. Vo, B.-T. Vo, G. Battistelli, L. Chisci, Consensus labeled random finite set filtering for distributed multi-object tracking, arXiv:1501.01579 (2015).
[38] B. Wei, B. Nener, Consensus labeled multi-bernoulli filtering for distributed space debris tracking, in: Control, Automation and Information Sciences (ICCAIS), 2017 International Conference on, IEEE, 2017, pp. 203–208.
[39] L. Pao, Centralized multisensor fusion algorithms for tracking applications, Control Eng. Pract. 2 (5) (1994) 875–887.
[40] L.Y. Pao, L. Trailovic, The optimal order of processing sensor information in sequential multisensor fusion algorithms, IEEE Trans. Automat. Contr. 45 (8) (2000) 1532–1536.
[41] L. Trailovic, L.Y. Pao, Computing budget allocation for efficient ranking and selection of variances with application to target tracking algorithms, IEEE Trans. Automat. Contr. 49 (1) (2004) 58–67.
[42] H.A. Blom, Y. Bar-Shalom, The interacting multiple model algorithm for systems with markovian switching coefficients, IEEE Trans. Automat. Contr. 33 (8) (1988) 780–783.
[43] W. Li, Y. Jia, Consensus-based distributed multiple model ukf for jump markov nonlinear systems, IEEE Trans. Automat. Contr. 57 (1) (2011) 227–233.
[44] J.L. Williams, Hybrid poisson and multi-bernoulli filters, in: Information Fusion (FUSION), 2012 15th International Conference on, IEEE, 2012, pp. 1103–1110.
[45] S. Geman, D. Geman, Stochastic relaxation, gibbs distributions, and the bayesian restoration of images, IEEE Trans. Pattern Anal. Mach. Intell. PAMI-6 (6) (1984) 721–741.
[46] G. Casella, E.I. George, Explaining the Gibbs sampler, Am. Stat. 46 (3) (1992) 167–174.
[47] B.-N. Vo, B.-T. Vo, H.G. Hoang, An efficient implementation of the generalized labeled multi-bernoulli filter, IEEE Trans. Signal Process. 65 (8) (2017) 1975–1987.
[48] R.G. Gallager, Stochastic Processes: Theory for Applications, Cambridge University Press, 2013.
[49] S. Brooks, A. Gelman, G. Jones, X.-L. Meng, Handbook of Markov Chain Monte Carlo, chapter 5, CRC press, 2011.
[50] Y. Bar-Shalom, X.R. Li, T. Kirubarajan, Estimation with Applications to Tracking and Navigation: Theory Algorithms and Software, John Wiley & Sons, 2004.
[51] F. Garin, L. Schenato, A survey on distributed estimation and control applications using linear consensus algorithms, in: Networked control systems, Springer, 2010, pp. 75–107.
[52] G. Battistelli, L. Chisci, F. Papi, A. Benavoli, A. Farina, A. Graziano, Optimal flow models for multiscan data association, IEEE Trans. Aerosp. Electron. Syst. 47 (4) (2011) 2405–2422.
[53] S. Deb, K.R. Pattipati, Y. Bar-Shalom, A multisensor-multitarget data association algorithm for heterogeneous sensors, IEEE Trans. Aerosp. Electron. Syst. 29 (2) (1993) 560–568.
[54] S. Deb, M. Yeddanapudi, K. Pattipati, Y. Bar-Shalom, A generalized sd assignment algorithm for multisensor-multitarget state estimation, IEEE Trans. Aerosp. Electron. Syst. 33 (2) (1997) 523–538.
[55] S. He, H.-S. Shin, A. Tsourdos, Track-oriented multiple hypothesis tracking based on tabu search and Gibbs sampling, IEEE Sens. J. 18 (1) (2017) 328–339.
[56] I. Hwang, C.E. Seah, S. Lee, A study on stability of the interacting multiple model algorithm, IEEE Trans. Automat. Contr. 62 (2) (2017) 901–906.
[57] D. Schuhmacher, B.-T. Vo, B.-N. Vo, A consistent metric for performance evaluation of multi-object filters, IEEE Trans. Signal Process. 56 (8) (2008) 3447–3457.

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Distributed multiple model joint probabilistic data association with Gibbs sampling-aided implementation

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